Statistical Failure Analysis of C3 Howitzer Barrels

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ABSTRACT

The C3 Howitzer fleet of the Canadian Army is ageing, and a number of Howitzer barrels have cracked at usage levels well below their rated lifetime. However, the majority of the Howitzer barrels remain uncracked creating a situation with very limited information from which to statistically determine if the observed cracks are compatible with the rated lifetime. A failure analysis is performed based on a Weibull failure model. A Markov Chain Monte Carlo technique is used to simulate model parameters via Bayesian inference to produce an accurate statistical distribution of possible model configurations given the limited available data. This method provides insight into the dynamics of the failure process, such as the expected lifetime of the Howitzer barrels and enables simulations of future failures given observed usage trends. Visualisations of the geographical and usage information of the Howitzer fleet combined with the model predictions provide insights to fleet managers and motivate recommendations. Bayesian methods presented including model building, prior construction and Hamiltonian Monte Carlo simulation are widely applicable to OR&A for modelling, forecasting and simulation. Finally, results are compared to those alternatively obtained from the STAN modelling language implementation in R.

Keywords: Cognitive Superiority; Markov Chain; Failure Analysis; Bayesian Inference; Statistical Simulation

1.0 INTRODUCTION

The Canadian Army maintains a fleet of approximately 100 C3 Howitzers. Upon a recent inspection of the fleet, roughly 10% of the cannon tube barrels were observed to have a crack and each cracked tube belonged to a Howitzer that had less accumulated usage than the rated lifetime claimed by the manufacturer. Since only a small fraction of the cannon tubes have been observed to have cracked it is impossible to determine definitively what the average lifetime will be once the entire population of tubes has failed. X-Ray diffraction studies and Finite Element Analyses have been performed to examine the root cause of the crack failure process [1]. These methods cannot however answer broader questions about the failure rate of the fleet.

The purpose of this study is to perform an analysis making use of statistical simulation methods to determine probabilistic answers to questions surrounding the lifetimes of the cannon tubes. These methods provide useful answers to questions about the cannon tube population that would otherwise be challenging to address given the limited available statistics. This conference proceedings paper is a condensed version of a report written for the Materiel management division of the Canadian Armed Forces [2], which was an update to a previous study [3].

1.1 Cannon Tube Usage Data

The results of this study are derived from two available crack inspection datasets, one compiled in 2014 and another compiled in 2020. The population consists of 105 cannon tubes, both the 2014 and 2020 datasets include the usage of each tube measured in units of Equivalent Full Charge (EFC) and the cracked or uncracked status of each tube. The rated lifetime of the cannon tubes is 7500 EFCs. Of particular interest to this study is a prediction of the true average lifetime of the cannon tube population, the Meant Time to Crack Formation (MTTCF).

The 2014 dataset also includes the number of rounds fired by each tube and the expected future usage rate of each tube. Since a round can be propelled with varying combinations of explosive charge bags, one round does not equal one EFC in general.

Figure 1 (left) shows the accumulated EFC versus rounds fired for each cannon tube from the 2014 dataset. Tubes #102 and #189 are observed to have accumulated a large number of EFCs from a relatively low number of fired rounds. This is a result of a difference in usage patterns for these two tubes compared to the general tube population, the rate of fire and intervals between barrel cleaning is substantially different. It was therefore decided to exclude these two tubes from this analysis leaving a population size of 103 cannon tubes.

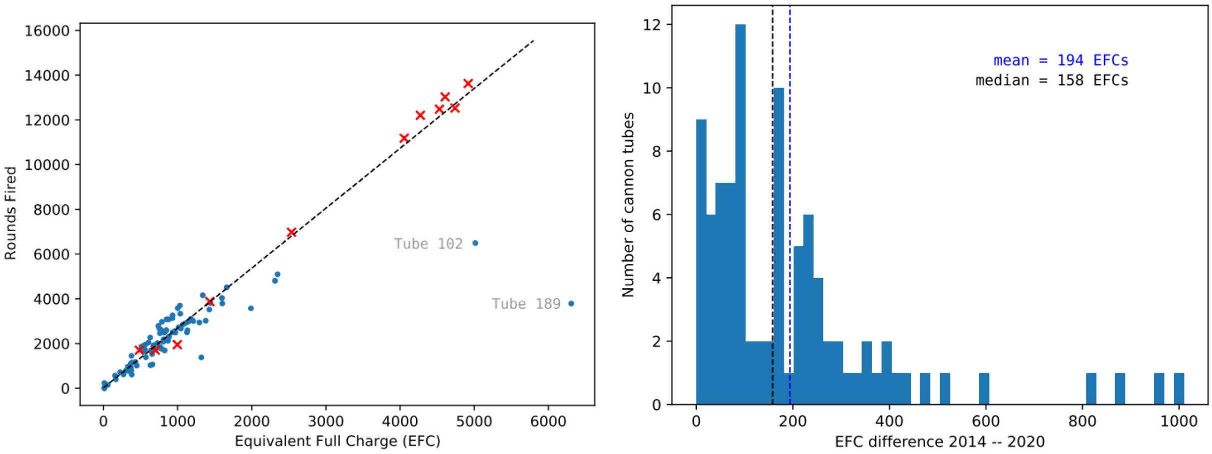

Figure 1: (Left) Crack inspection results by total rounds fired and total accumulated EFCs for the population of 105mm cannon tubes from the 2014 data. Blue circles (red crosses) denote tubes which were uncracked (cracked) at the time of inspection, respectively. Tubes #102 and #189 are highlighted as outliers to the prevailing linear relationship. (Right) Distribution of the number of EFCs fired between 2014 and 2020 for the population of cannon tubes.

There were 11 cracked tubes identified in the 2014 inspection. The 2020 inspection revealed that one new tube (#47) had cracked and one tube (#34) which was believed to have been cracked in 2014 is not in fact cracked and is currently in service. The accumulated usage between the two inspections is presented in Figure 1 (right) this includes the usage of the 91 uncracked tubes as well as tube #47. It is clear from the distribution that the usage varies dramatically across the tube population. The median total lifetime usage of the uncracked tube population in 2020 was 945 EFCs (the median usage in 2014 was 780 EFCs). During the six-year period, the most used tube accumulated 1010 EFCs. The median tube accumulated 158 EFCs over the period and twenty tubes accumulated less than 50 EFCs. Overall, the usage rate of the average tube is slow compared to the rated lifetime of the tubes, therefore we can expect the failure process of the fleet to play out over many decades as usage is slowly accumulated across the fleet.

1.2 Cannon Tube Situational Data

The 2020 inspection dataset includes, in addition to the EFC and cracked status of the cannon tube population, the location and unit information for each tube. With this information, one can look for differences in the observed outcomes with respect to where and by whom the tubes are used. Combining the location information with the accumulated EFC data, Figure 2 shows the distribution of cannon tube EFCs coloured to show either the location or whether the tube is operated by a regular or reserve force unit.

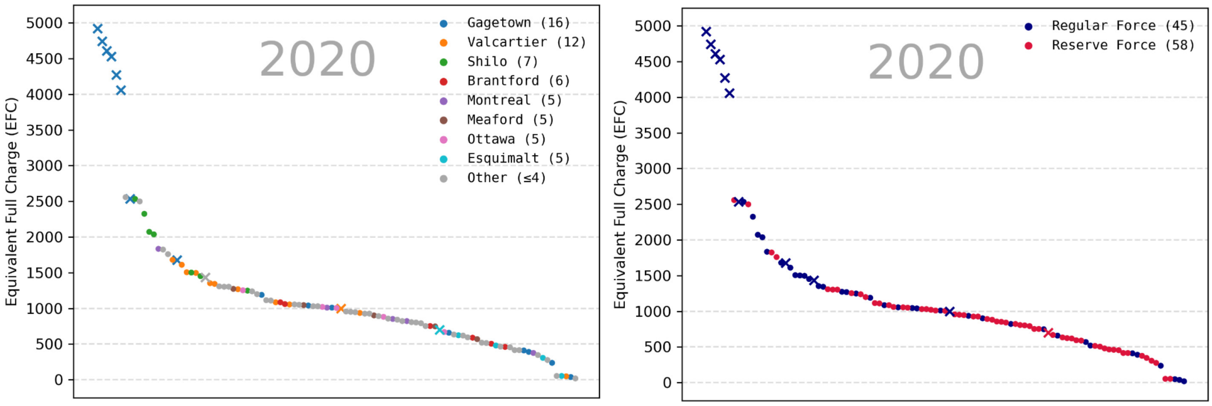

Figure 2: Distribution of cannon tube total accumulated EFCs from the 2020 inspection dataset. A cross or circle indicates a cracked or uncracked tube, respectively. (Left) Locations with five or more tubes are identified with a unique colour and all other locations with four or less tubes are shown in grey. (Right) Tubes that are operated by a regular or reserve force are shown in blue or red, respectively. The horizontal axis in these figures are an index that orders the tubes from most to least usage in EFCs.

Six heavily used (>4000 EFC) cracked tubes are in Gagetown, NB with a regular force unit, as well as two other cracked tubes. Also of interest are Shilo, MB and Brandon, MB, where the combined ten tubes have an average usage of over 1900 EFCs and no cracks have been observed. Several of these Shilo/Brandon tubes are used for Avalanche control (AVCON) operations in Rogers Pass, BC. This AVCON operation is a relatively intensive use case for the C3 howitzers and as a result, four out of the five most used tubes during the six-year period between inspections were located in Shilo. Thus, the observed positive outcome of no cracks in these AVCON tubes is notable and worthy of further study into the maintenance and usage practices of this subset of the cannon tube population. Nearly all of the cracked tubes were operated by regular force units, likely because the regular force tubes have higher average accumulated usage. If the distribution of cracks were more evenly spread between regular and reserve force tubes it would we worthwhile to study the two populations separately, however given the current distribution it is preferable to study the population as a whole to take advantage of all available statistics.

2.0 METHOD

The time to crack formation is modelled with the Weibull distribution, a widely used distribution in failure analysis [4]. This distribution has a shape parameter (ν) and a scale parameter (η) and is flexible enough to model both an *infant mortality* type failure process where the failure rate decreases over time ($\nu < 1$) and a *wear-out* type failure process where the failure rate increases over time ($\nu > 1$). The scale parameter of the distribution relates to the observed lifetimes of the population, the larger the predicted scale parameter the longer the expected lifetime. Given our observed data, consisting of the usage and cracked status of each tube, the goal of the analysis is to determine the *likelihood* of obtaining these observations as a function of the Weibull parameters.

The failure and survival functions of the Weibull distribution define the probability that a crack has or has not formed as a function of number of EFCs accumulated, respectively. Given the observed data, these functions are used to construct a single *Likelihood Function* of the data that depends on the two Weibull parameters. Figure 3 (left) shows the contours of this likelihood function. The inner most contour represents the most likely area, this indicates that the data is most compatible with a wear-out type failure process ($\nu > 1$) with a scale (lifetime) of 3000 to 4000 EFCs.

The likelihood function is one of two key inputs to the *Markov Chain Monte Carlo* (MCMC) method. MCMC methods are a class of algorithm designed to sample from a probability distribution. The likelihood function represents the data and a simulation generated based on this function alone would be initially biased towards the observed data. In order to unbias the simulation an input called the *prior probability distribution* or simply a *prior* is employed. The prior serves as a baseline for the simulation that can be constructed to satisfy initial criteria. For this study, the criteria are 1) that the rated lifetime is 7500 EFCs; and 2) that the failure process could be either a wear-out or an infant mortality type process. The prior used in the simulation has an equal probability assigned to the infant mortality case (50% $\nu < 1$) and the wear out case (50% $\nu > 1$). The prior distribution also has equal probability that the MTTCF is less than the rated lifetime and greater than the rate lifetime. The prior is constructed from the *gamma* distribution, chosen because it is the *conjugate prior* of the likelihood function and therefore has ideal properties for Bayesian inference [5]. Figure 3 (right) shows the contours of the likelihood function combined with the prior.

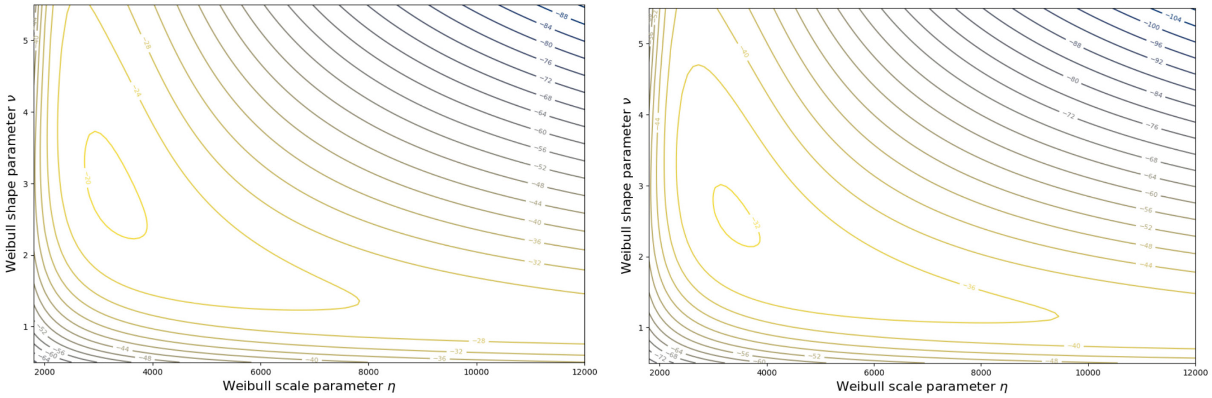

Figure 3: Contour plot of the likelihood function (left) and likelihood plus prior distribution (right) in the Weibull parameter space. The inner contours represent the areas of parameter space most compatible with the observed data. The prior acts to reshape the contours towards the rated lifetime assumption at higher η.

The MCMC takes as input the likelihood function and prior distribution and generates a statistical sample of possible pairs of Weibull parameters (shape and scale). The MCMC process proceeds via the Metropolis-Hastings (MH) algorithm [6]. The MH algorithm starts at a chosen point in parameter space ($\nu = 1$, $\eta = 7500$ EFCs) and generates new candidate points iteratively via a random Gaussian walk. Candidate points are rejected or accepted into the sample randomly and weighted by their compatibility with the likelihood function. A sample was generated containing 5,000,000 pairs of Weibull parameters. The MH algorithm ensures that parameters that are more likely given the observed data will appear more frequently in the sample. The final (*posterior*) distributions of the parameters are used to make statistical predictions.

The MH algorithm is a frequently used and straightforward to implement method for generating the posterior sample via MCMC. It performs adequately here, as the contours of the likelihood function are smooth and compact in the two dimensional Weibull parameter space. In general, however, the random walk behaviour makes the MH algorithm inefficient in exploring the target distribution, especially in higher dimensions and with more complex target distributions. A modern alternative method, Hamiltonian Monte Carlo (HMC), borrows ideas from physics to explore distributions significantly more efficiently [7]. In this study, the R programming language is used to perform the HMC sampling using the Rstan interface to Stan [8], which provides full Bayesian inference using the no-U-turn sampler (NUTS) [9]. HMC is used as an independent crosscheck of the results, the same likelihood and prior functions described above are input to Stan, which implements the model and performs the sampling. The sample consists of 16,000 parameter pairs. Smaller necessary sample sizes are another advantage of the efficient exploration of HMC.

3.0 RESULTS

Figure 4 (left) shows the distribution of the simulated shape parameters. From this distribution, we conclude that the failure process of the C3 cannon tubes is likely a wear-out type process as 99.8% of the simulate shape parameters are greater than 1. The simulated sample is used to derive the posterior predictive distribution of the MTTCF shown in Figure 4 (right). This distribution is representative of the lifetime distribution of the cannon tube population. The peak of the distribution occurs at approximately 2835 EFCs and the 95% credible interval is [2104, 4602] EFCs. We observe that 99.4% of the simulated MTTCF values are less than the rated lifetime. Compatible results are obtained from the sample generated via HMC implemented in Stan; the result comparison is summarized in Table 1. The key results, that the process is wear-out failure and that the true MTTCF is likely less than the rated lifetime are confirmed by the HMC results. The MTTCF distribution from HMC is more sharply peaked at 3700 EFCs centred on higher EFC values. This suggests that the prior distribution, which will tend to pull the distribution towards the rated lifetime of 7500 EFCs has more weight in the HMC implementation.

Table 1: Comparison of key results derived from MCMC samples generated by the standalone implementation of the Metropolis-Hastings sampler and from the default Hamiltonian Monte Carlo sampler implemented in Rstan. All generated shape parameters from Rstan were greater than one indicating a 100% likelihood of a wear out process.

Result	Standalone MH sample	Rstan HMC NUTS sample
Likelihood of a wear out type failure process	99.8%	100%
Likelihood that the MTTCF is less than the rated lifetime of 7500 EFCs	99.4%	99.9%
MTTCF mode, median (EFC)	2835, 3057	3700, 3885
MTTCF credible intervals (EFC)	80% CI = [2321, 3625] 95% CI = [2104, 4602]	80% CI = [3242, 4395] 95% CI = [3014, 4959]

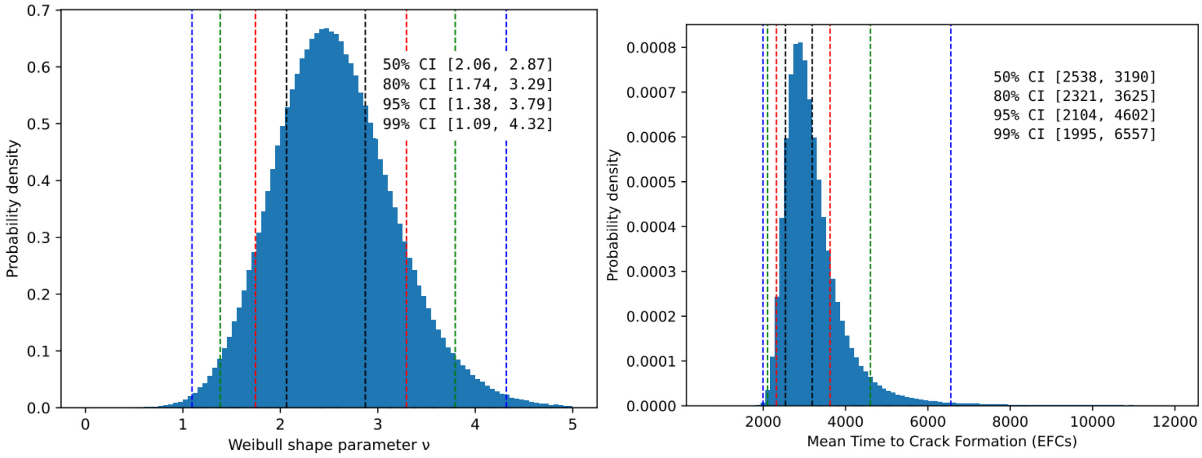

Figure 4: Posterior probability distribution of the Weibull shape parameter (left) and MTTCF (right) in units of EFC. A shape parameter greater than one indicates a wear-out type failure process. CI stands for Credible Interval indicating the range of values that contain a given fraction of the distribution.

The MTTCF distribution of Figure 4 is indicative of the lifetime of an unused cannon tube. For each tube, the simulated sample is used to generate a *residual lifetime distribution* to estimate the remaining lifetime. The median of this distribution is the amount of usage that results in a 50% chance of a new crack. Since the two datasets provide EFCs for the same population of cannon tubes, the average usage over the six-year period is known. The remaining lifetime distributions are used to predict future failures, assuming future usage will follow the observed trend. This method predicts that two tubes have a 50% chance of failure within the next ten years and another within the next fifteen years. Of note, is that all three tubes are used for AVCON, which has produced no cracked tubes despite the high EFC load. Also of note are the 22 cannon tubes that have accumulated less than 10 EFCs per year between the two inspections. It is evident that there is a wide range of usage rates across the cannon tube population and an effort to more evenly distribute the EFC load across the fleet would likely decrease the crack formation rate.

4.0 CONCLUSIONS

There remains a gap in the data between 2500 to 4000 EFCs. All tubes with greater than 4000 EFCs are cracked and the highest usage uncracked tube has 2563 EFCs. Several of the tubes located in Brandon and Shilo have accumulated more than 2000 EFCs and are potentially useful for improving our understanding of crack formation in the range between 2500 to 4000 EFCs. On the other hand if the goal is to preserve the fleet and minimize future cracking, an effort to reduce the future usage of these high usage tubes is recommended.

An effort to compile failure data for all similar Howitzer fleets across the NATO allies would provide statistical power to increase the precision of future results. If the sample size were sufficiently large, categorical variables could be introduced into the analysis to understand how the situational information (who is using the Howitzers and for what purpose?) influences the failure process. The investigation into the more powerful simulation toolset available in Stan will be useful if additional complexities are added to the model. In summary, both the standalone and Stan MCMC methods predict a >99% likelihood that the failure process is a wear-out type process and that the MTTCF is less than the rated lifetime of 7500 EFCs.

5.0 REFERENCES

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